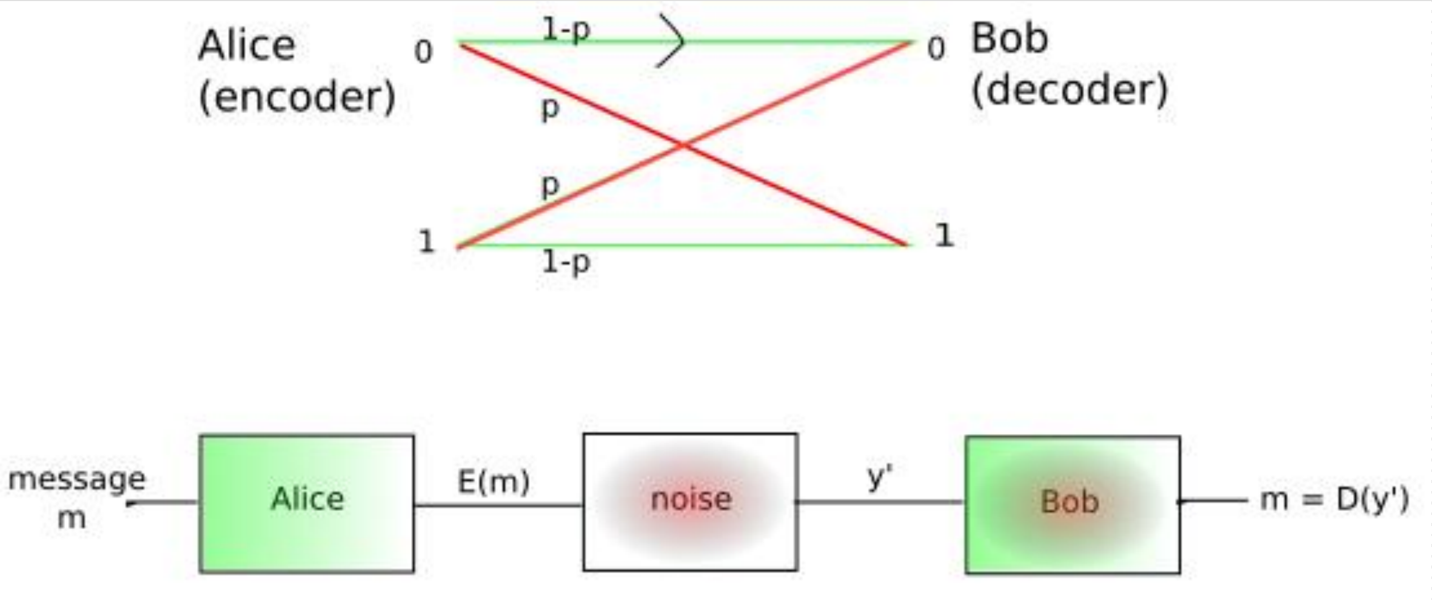
Channel Coding

Recap…

- Information is transmitted through channels (eg. Wires, optical fibres and even air)
- Channels are noisy and we do not receive what was transmitted

System Model

- A Binary Symmetric Channel



- Crossover with probability p

Repetition Coding

$$0 \rightarrow 000$$

$$1 \rightarrow 111$$

- Assume 1/3 repetition
- What is the probability of

error ?

$$P_e = {}^3C_2 p^2 (1 - p) + p^3$$

- If crossover probability p = 0.01, Pe ≈ 0.0003
- Here coding rate R = 1/3. Can we do better? How much better?

Shannon’s Theorem

- Given,
  - A noisy channel (some fixed p)
  - A value of Pe which we want to achieve

“We can transmit through the channel and achieve this probability of error at a maximum coding rate of C(p)”

- Is it counterintuitive?
- Do such good codes exist?

Channel Capacity

- C(p) is called the channel capacity
- For binary symmetric channel,

$$C(p = 0.01) = 0.9192$$

- Can we really design codes that achieve this rate? How?

Parity Check Codes

- #information bits transmitted = k
- #bits actually transmitted = n = k+1
- Code Rate R = k/n = k/(k+1)

- Error detecting capability = 1
- Error correcting capability = 0

2-D Parity Check

- Rate?
- Error detecting capability?
- Error correcting capability?

1	0	0	1	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1

Last column consists of check bits for each row

Bottom row consists of check bit for each column

Linear Block Codes

- #parity bits n-k (=1 for Parity Check)
- Message m = $\{m_1\ m_2\ \dots\ m_k\}$
- Transmitted Codeword c = $\{c_1\ c_2\ \dots\ c_n\}$
- A generator matrix G_{kxn}

$$c = mG$$

- What is G for repetition code?
- For parity check code?

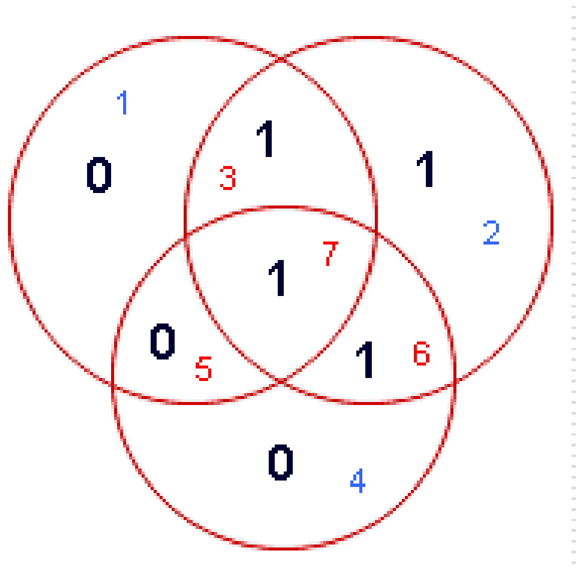
Linear Block Codes

- Linearity

$$c_1 = m_1 G,$$
$$c_2 = m_2 G$$

$$c_1 \oplus c_2 = (m_1 \oplus m_2)G$$

- Example : 4/7 Hamming Code
  - k = 4, n = 7
  - 4 message bits at (3,5,6,7)
  - 3 parity bits at (1,2,4)
  - Error correcting capability =1
  - Error detecting capability = 2
  - What is G?



Cyclic codes

- Special case of Linear Block Codes
- Cyclic shift of a codeword is also a codeword
  - Easy to encode and decode,
  - Can correct continuous bursts of errors
  - CRC (used in Wireless LANs), BCH codes, Hamming Codes, Reed Solomon Codes (used in CDs)

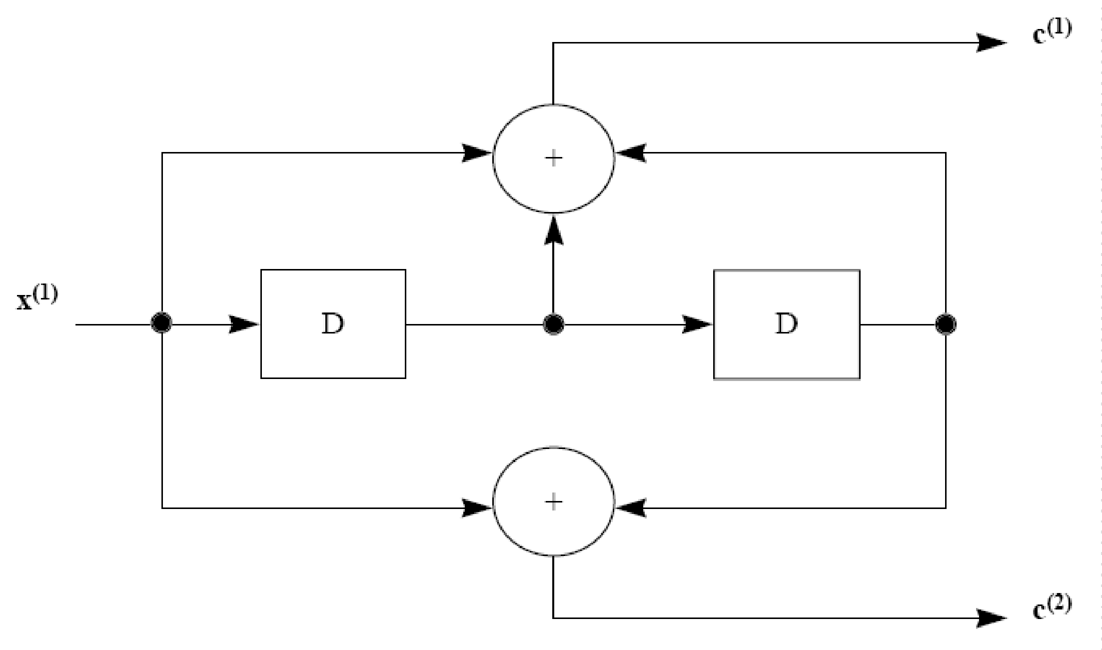
Convolutional Codes

- Block codes require a buffer
- What if data is available serially bit by bit? Convolutional Codes
- Example

k = 1

n = 2

Rate R = ½



Convolutional Codes

- Encoder consists of shift registers forming a finite state machine
- Decoding is also simple – Viterbi Decoder which works by tracking these states
- First used by NASA in the voyager space programme
- Extensively used in coding speech data in mobile phones

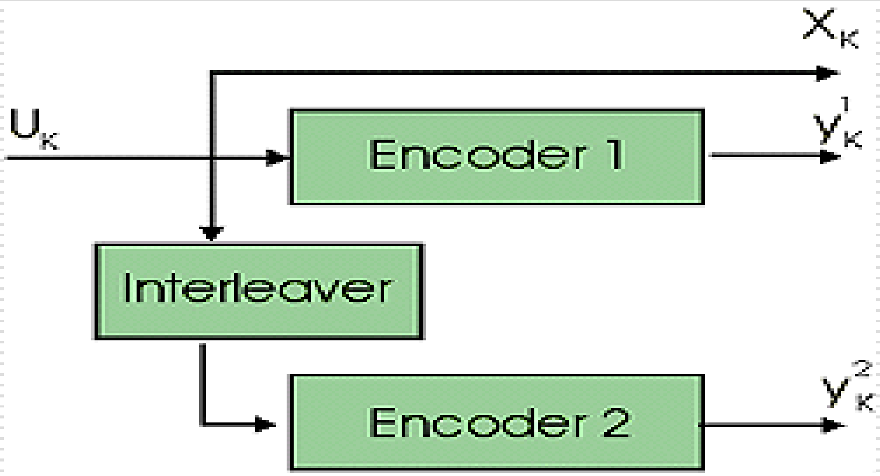
Achieving Capacity

- Do Block codes and Convolutional codes achieve Shannon Capacity?

  Actually they are far away
- Achieving Capacity requires large k (block lengths)
- Decoder complexity for both codes increases exponentially with k – not feasible to implement

Turbo Codes

- Proposed by Berrou & Glavieux in 1993



- Advantages
  - Use very large block lengths
  - Have feasible decoding complexity
  - Perform very close to capacity
- Limitation – delay, complexity

Summary

- There is a limit on the how good codes can be
- Linear Block Codes and Convolutional Codes have traditionally been used for error detection and correction
- Turbo codes in 1993 introduced a new way of designing very good codes with feasible decoding complexity